tors, the hyperbolic trapezoids (the parallel sides being parallel to one of the asymptotes, the other two sides being the hyperbolic arc and the other asymptote), but he finds this less general and less convenient. Nowhere, either in eighteenth century or nineteenth century authors have we been able to find a reference to Karsten's geometric construction of imaginary logarithms.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

374. Proposed by H. PRIME, Boston, Massachusetts.

Divide an angle of 30° into two parts so that the product of the third and fourth powers of their sines (or cosines) shall be a maximum. To be solved without using the methods of calculus. [From *The Maine Farmers' Almanac*, 1912.]

Solution by H. E. TREFETHEN, Colby College.

Let x and $30°-x$ be the two parts. $\sin^4 x \sin^3(30°-x)=\text{maximum}$, when $\sin^{\frac{4}{3}} x \sin(30°-x)=\text{maximum}$, or when $\sin^{\frac{4}{3}} x \cos x - \sin^{\frac{7}{3}} x \sqrt{3}=\text{maximum}$. Put $\sin^{\frac{1}{3}} x=y$, $\cos x=(1-y^6)^{\frac{1}{2}}$; and then $y^4(1-y^6)^{\frac{1}{2}} - y^7\sqrt{3}=m$, whence

$$y^{14}-y^8/4+my^7\sqrt{3}/2+m^2/4=0\ldots(1).$$

Let c be a value of y that renders m a maximum. Then the first member of (1) must be exactly divisible *twice* by $y-c$, since a maximum or minimum corresponds to two equal roots. The quotient is readily written down by the synthetic process. The first remainder $=c^{14}-c^8/4+mc^7\sqrt{3}/2+m^2/4=0$, since the division must be exact. Also the second remainder $=14c^{13}-2c^7+7mc^6\sqrt{3}/2=0$.

Eliminating m from the last two equations we get $196c^{14}-203c^8+16c^2=0$. Whence $c^2=0$ corresponding to a minimum, and $196c^{12}-203c^6+16=0$, $c^6=y^6=\sin^2 x=4.813227/56$, or $53.186773/56$. The former gives $x=17° 2' 52.9''$; the latter ($=\cos^2 x$) gives $x=12° 57' 7.1''$, and $30°-x=12° 57' 7.1''$ or $17° 2' 52.9''$.

It is to be noted that this method for determining maxima and minima can be applied to *any algebraic* expression to which the methods of calculus are applicable.

Also solved by J. Scheffer and A. H. Holmes.